I. Introduction

Recent theoretical and experimental investigations of dust-contaminated plasmas (dusty plasma, DP) [1] have established the existence of strongly coupled DP lattices (crystals). These crystalline configurations, consisting of highly charged massive dust grains, are typically formed in the sheath region above a horizontal negatively biased electrode in gas discharge experiments (e.g. [1, 2]). Typical low-frequency oscillations are known to occur in [1, 2] in these mesoscopic dust grain quasi-lattices in the longitudinal (in-plane, acoustic mode), horizontal transverse (in-plane) and vertical transverse (off-plane, inverse dispersive optic-like mode) directions. A variety of 2D and 3D configurations are possible [16], although the spontaneous occurrence of sequential 2D layers seems to be the most often encountered possibility. A 1D DP crystal has also been realized experimentally, by using appropriate substrate potentials. Such 1D lattices have been shown to host collective excitations, in the form of solitons, localized envelope wavepackets, as well as discrete breather-type excitations (see [8] and Refs. therein).

In the present work a hexagonal DP lattice in considered. Transverse motion in this system is described by a Klein-Gordon-like Hamiltonian in the presence of an asymmetric quartic potential. By adopting real values for the potential (nonlinearity) parameters, as provided by experiments [3, 4, 5], and using the results of [6, 7], we shall prove that 2D DP crystals may support single-site as well as multi-site localised oscillations (multiblenders) [9].

II. Existence of Multiblenders in a Hexagonal Lattice

We consider the hexagonal lattice of fig. 1, on site nonlinear potential $V(x)$ and nearest neighbor linear interaction, with coupling parameter $c$.

This system is described by a Klein-Gordon Hamiltonian of the form of eq. 1.

$$H = H_0 + iH_1 = \sum_{\tau} |\psi_\tau|^2 + \sum_{\tau} V(x_\tau) + \sum_{\tau} \left[ (x_{\tau+1} - x_\tau)^2 + (\sin\theta_\tau - \sin\theta_\tau + \xi) \right]$$

In [6] it is proven that, if the anharmonicity and the nonresonance with the linear spectrum conditions hold, this system supports multi-site breathers if

$$\frac{\partial^2 H_1}{\partial \phi^2} |_{\phi = 0} = 0 \quad (\text{2})$$

and

$$\frac{\partial^2 H_1}{\partial \phi \partial \theta} |_{\phi = 0} = 0 \quad (\text{3})$$

where $H_{1,0}$ is the average value of $H_1$ along the unperturbed periodic orbit, $t$ is the period of the resulting breather, and $\phi = \omega x - \omega t, \theta = \theta_0 + \xi$ are the phase differences between the oscillators.

Since, the solution for a single oscillator can be an even function of the angle of the motion we can write

$$s(x) = \sum_{a,b} s(x_0) \cos(\omega x_0 + \theta)$$

Then condition (2) becomes

$$\frac{\partial^2 H_1}{\partial \phi^2} |_{\phi = 0} = 0$$

which have always at least the solutions

$$\phi = 0, \phi = \pi$$

so, up to 3 moving oscillators we can distinguish the following 4 cases.

Case (a): Single site breather

Case (b): $\phi = \pi = 0$ In-phase breather

Case (c): Out of phase breather

Case (d): $\phi = \pi = \theta / 3$ Vortex breather

III. The dusty plasma crystal

For negligible damping, and after the necessary normalization, the vertical, out of plane displacement in a DP crystal can be described by a Hamiltonian of the form (1) with $c = 0$ in account of the inverse dispersion of the lattice. The substrate potential can be approximated by a polynomial of the form

$$F(x) = \frac{x^2}{2} + \frac{x^4}{4} + bx$$

In a personal communication, Prof. Melzer suggested the set of values for $a,b,c$ which are shown in Table 1.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

IV. Conclusions – Future Work

We have shown that a two-dimensional dusty plasma crystal can support single-site breathers, and in-phase 3-site breathers, while the rest of the theoretically predicted multi-site breathers are highly unstable. These results will hopefully be confirmed by appropriate experiments.

References

[11] Case I of Table 1 which the parameter values suggested by A. Melzer (personal communication)